

# Preparation of atomic velocities by bound-state to resonance conversion

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A procedure is proposed to control the average and width of the velocity distribution of ultra-cold atoms. The atoms are set initially in a bound state of an optical trap formed by an inner red detuned laser and an outer blue detuned laser. The bound state is later converted into a resonance by a suitable change of the laser intensities. An optimal time dependence of the switching process, between the sudden and adiabatic limits, adjusts the final translational energies to the Lorentzian shape of the resonance state.

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## I. INTRODUCTION

With the advent of laser cooling techniques, the traditional velocity selection or preparation methods [1] have to be substituted, due to the increasing importance of gravity and the quantum nature of translational motion, by new methods based on Doppler sensitive stimulated Raman transitions [2, 3], coherent population trapping into a dark state [4], or Bragg diffraction by a moving, periodic, optical potential [5, 6, 7, 8, 9]. To complement these methods and overcome some of their limitations [9], it is worthwhile to explore other approaches based on different physical mechanisms.

Fabry-Perot (FP) matter-wave interferometers realized with detuned lasers or microwave cavities have been also proposed to provide coherent atomic velocity selection or trapping [10, 11, 12, 13, 14]. Moreover, the transmission behavior of a Bose-Einstein condensate [15, 16] through a double barrier in a waveguide has been described in [17], and through an optical lattice in [18]. In a recent paper [11], we have explored the fundamental limits of a matter-wave Fabry-Perot optical device made of two blue-detuned laser barriers and a red-detuned laser well, for selecting both the average and the width of the atomic velocity distribution. The basic control knob was the well-depth, which lets modify the resonance energy. It was theoretically and numerically demonstrated that this method may produce arbitrarily small velocities but, since it is based on filtering the incident velocity distribution with a resonance peak of the transmission probability, the resulting fraction of transmitted atoms may be very low and will depend strongly on the incident state.

The present work describes a modified approach aimed at a more efficient preparation and control of the average and width of the final velocity distribution. In common

with the transmission resonance method, it may also be implemented with red and blue detuned lasers, a red detuned laser forming an inner well and a blue detuned laser for an outer barrier. The working principle is now that the atom is initially prepared in a bound state. Then, the bound state is converted into a resonance by a suitable change of the laser intensities, so the atom will leak out and move asymptotically with the desired velocity distribution. This procedure will be described in detail in Section II. A necessary requirement for our method to be of any practical use is the knowledge of the dependence between laser parameters and the resonances. This relation will be examined in Sec. III. An optimal time dependence of the switching process, between the sudden and adiabatic limits, will be described in Section IV which adjusts the asymptotic kinetic energies to the Lorentzian shape of the resonance state. Final conclusions and comments will be provided in Section V.

## II. BOUND-STATE TO RESONANCE CONVERSION

For simplicity, we shall assume a one dimensional (1D) model corresponding to the effective 1D atomic motion in a narrow waveguide, and a “square” shape for each laser intensity, see Fig. 1, although similar results can be achieved for smoother profiles [11]. The infinite wall at the origin is also a simplifying feature of the model, but it is not strictly necessary. In particular, one could also use two finite barriers, one at each side of the well [11], to represent the radial potential profile of a cylindrical confinement with free atomic motion or weak confinement in the axial direction. At this stage we also assume a simple single-atom or independent-atoms framework described by the Schrödinger equation and disregard non-linear effects that could be incorporated within a mean-field treatment as in [11].

The starting point of the velocity preparation process is a laser configuration which holds only one bound state. It is assumed that the atom can be prepared in this

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ground state (see Fig. 1(a)). Several possibilities exist to prepare that initial state: for example, the original trap could hold more than one bound state; in that case an arbitrary trapped atomic state overlaps with several of them, but the trap may be modified to hold one bound state only so that the wave component in the continuum subspace is eliminated by its evolution away from the interaction region. More sophisticated and efficient methods without losing atoms may be based on ground-state cooling using resolved-sideband transitions [19]. Pushing up the potential well later on, the ground state will eventually become the only bound state, thus realizing our starting point objective.

Once the initial state of Fig. 1(a) is formed, the potential well is moved upwards by decreasing the intensity of the red-detuned laser, i.e.  $V_w$  is decreased. In addition, the intensity of the blue-detuned laser can also be changed. This is represented in Fig. 1(b), where the potential switch has been performed suddenly with respect to other relevant time scales. The consequence is that the bound state becomes, for a final well depth shallower than a threshold value, a “resonant state”. As it is well known, resonances may be regarded as quasi-bound states associated with poles of the  $S$ -matrix in the lower half-momentum plane; they can be linked continuously with bound states (poles on the positive imaginary axis) by varying the potential parameters. An important difference though, is that bound states are in Hilbert space and normalizable, while Gamow (resonant) states are not, since they increase exponentially at large distances from the potential center. The normalized state achieved by shifting the well bottom, as in Fig. 1(b), is thus not a true Gamow state, but it will share approximately some of its properties, in particular its decay rate, the basic Lorentzian shape in energy space and its coordinate-space form in the potential region. We insist that this agreement is necessarily a partial one.

After the switching process, the atom will leak out (see Fig. 1(c)) having a given (total) energy distribution. Note that the energy distribution calculated at the end of the switching process, i.e., at a time when the atom is still interacting with the trap, is equal to the kinetic energy distribution of the released atoms at asymptotically large time, as it follows from energy conservation. Therefore at a sufficient large time, the atom will move with the desired velocity distribution.

Two limits consisting on sudden or infinitely slow well switching may be considered: (a) A sudden well shift produces a state with contributions from higher resonances. They will lead to perturbations with respect to the ideal velocity distribution which will affect the short time decay behavior; (b) The opposite limit of very slow switching implies a different problem: since the pole motion in the complex momentum plane up to the final desired resonance position is slow, a continuum of other resonances are excited. They will have a decay time larger than the one desired, thus inducing a deviation with respect to the desired exponential decay rate, in this case due to a bias

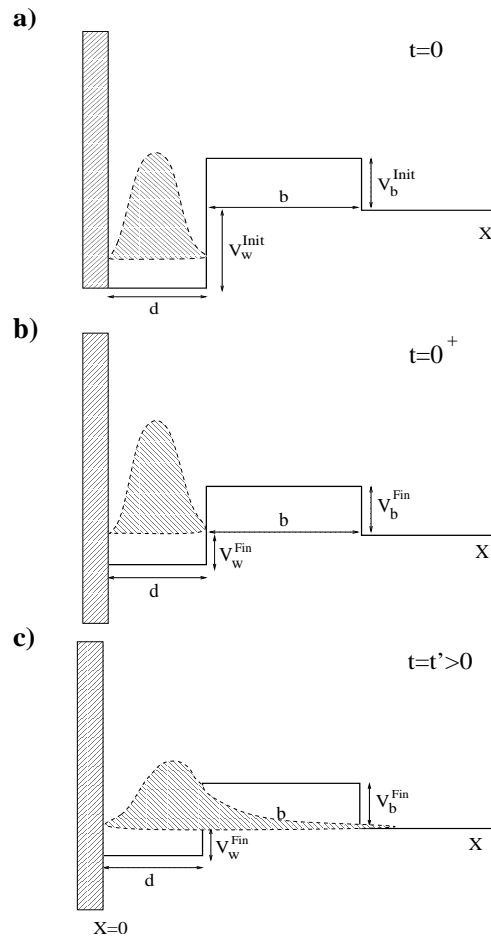


FIG. 1: Schematic representation of the velocity preparation method. The left barrier represents an “infinite wall”.

towards slow components. We will see in Section IV that an adjustment of the switching time may avoid the perturbations of the fast and the slow processes and produce an excellent agreement with the Lorentzian shape of the energy distribution.

An implementation of the proposed velocity preparation method will require the knowledge of the dependence between well/barrier parameters and the resonances such that the atom leaving the trap will have the desired velocity distribution. This dependence can be achieved experimentally or theoretically, as it will be described in the next section.

### III. CONFIGURATIONS AND CORRESPONDING RESONANCES

We consider the model based on a well and a barrier with variable depth/height represented in Fig. 1. The effective potential is assumed to take the initial and final

forms

$$V^{Init/Fin}(x) = \begin{cases} \infty & : x \leq 0 \\ -V_w^{Init/Fin} & : 0 < x \leq d \\ V_b^{Init/Fin} & : d < x \leq d+b \\ 0 & : x > d+b \end{cases}.$$

In the final configuration, see Fig. 1(b), the stationary states of a single ultra-cold atom moving along the  $x$  direction will satisfy

$$\left[ -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V^{Fin}(x) \right] \psi_k(x) = E_k \psi_k(x),$$

where  $E_k = \frac{\hbar^2 k^2}{2m}$ . For the calculations we have used the mass of  $^{23}\text{Na}$ . The scattering states will have the form

$$\psi_k(x) = \frac{1}{\sqrt{2\pi}} \begin{cases} C_1 e^{iqx} + C_2 e^{-iqx} & : 0 \leq x \leq d \\ C_3 e^{iq'x} + C_4 e^{-iq'x} & : d \leq x \leq d+b \\ e^{-ikx} - S(k) e^{ikx} & : x \geq d+b \end{cases},$$

where  $q = \sqrt{k^2 + 2mV_w/\hbar^2}$  and  $q' = \sqrt{k^2 - 2mV_b/\hbar^2}$  (for the rest of this section we omit the superscript  $^{Fin}$ ).  $q$  and  $q'$  have a branch cut in the  $p$ -plane joining the two branch points at  $\pm i\sqrt{2mV_w/\hbar^2}$  and  $\pm \sqrt{2mV_b/\hbar^2}$ , respectively. The different coefficients are obtained from the matching conditions at  $x=0$ ,  $x=d$ , and  $x=d+b$ . The resonances and bound states can be calculated from the poles of the  $S$ -matrix in the complex  $k$ -plane. They are solutions of the equation

$$\Omega(k) := -(k - q') [q + e^{2idq}(q - q') + q'] + e^{2ibq'}(k + q') [q - q' + e^{2idq}(q + q')] = 0.$$

The corresponding roots in the upper half-imaginary axis are the bound states of the system, while the roots in the fourth and third quadrant are resonances and antiresonances, respectively.

An alternative way to find the resonances is to look for jumps of the phase shift  $\delta(k)$ ,

$$\delta(k) = \frac{1}{2i} \log [S(k)],$$

or the peaks of the Wigner delay time  $\Delta t$ ,

$$\Delta t(k) = 2\hbar \frac{\partial \delta(E_k)}{\partial E_k} = \frac{2m}{\hbar k} \frac{\partial \delta(k)}{\partial k}. \quad (1)$$

This may be easier than determining the poles by analytical continuation of  $S(k)$ . In the Breit-Wigner regime of isolated and sharp resonances, the information about the resonances given by these peaks may be straightforwardly related to the poles in the complex  $k$ -plane. If  $E_{res} = E_R - i\Gamma/2$  is the complex energy obtained from the phase shift (central position  $E_R$  and width at half height  $\Gamma$ ), then

$$E_R = \frac{\hbar^2}{2m} (k_1^2 - k_2^2), \quad \Gamma = \frac{2\hbar^2}{m} k_1 k_2,$$

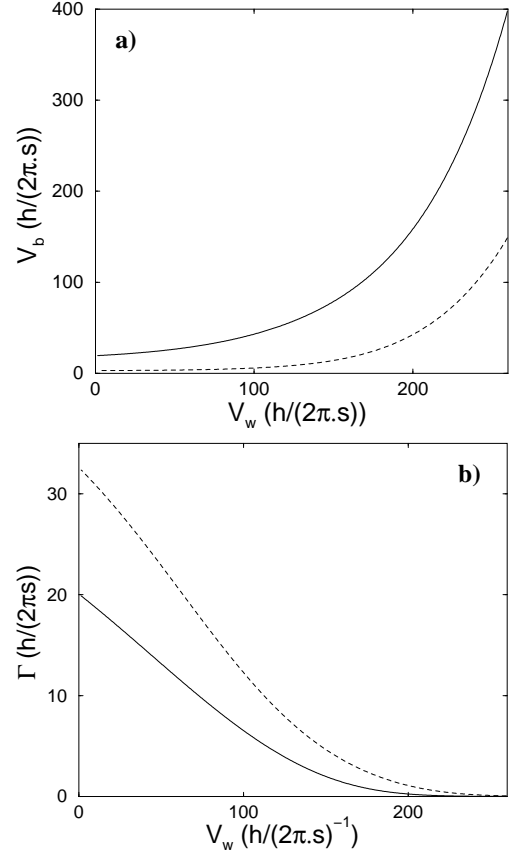


FIG. 2: Barrier height  $V_b$  versus well depth  $V_w$  (upper panel) and, energy width  $\Gamma$  versus  $V_w$  (lower panel) for constant resonance energies  $E_R$ :  $E_R = 53.391\hbar/s$  (solid lines) and  $E_R = 7.422\hbar/s$  (dashed lines).  $b = 10\mu\text{m}$ ,  $d = 5\mu\text{m}$ .

where  $k_{res} = k_1 + ik_2$  is the corresponding complex wave-number for which  $\Omega(k_{res}) = 0$ .

Different combinations of well depth  $V_w$  and barrier height  $V_b$  lead to the same resonant energy  $E_R$ . In Fig. 2(a) we have plotted the curves for two different resonant energies  $E_R$  with  $b = 10\mu\text{m}$ , and  $d = 5\mu\text{m}$ . Along a curve for a fixed  $E_R$ , the resonance width will change. This change is plotted in Fig. 2(b). Notice that for small values of the well depth  $V_w$ , the barrier height  $V_b$  that keeps the resonant energy  $E_R$  fixed is almost constant. In contrast,  $\Gamma$  changes drastically for small changes of  $V_w$  in shallow wells.

#### IV. RESULTS FOR DIFFERENT SWITCHING TIMES

Let  $\varphi_0(x)$  be the ground state in the initial potential configuration of Fig. 1(a) with  $V_w^{Init} = 350\hbar/s$ ,  $V_b^{Init} = 400\hbar/s$ . For the destination resonance in Fig. 1(b), we choose the lowest one corresponding to  $V_w^{Fin} = 100\hbar/s$  and  $V_b^{Fin} = 200\hbar/s$ . In this case, the resonant complex energy is  $E_{res} = (134.509 - i1.217)\hbar/s$ . To characterize

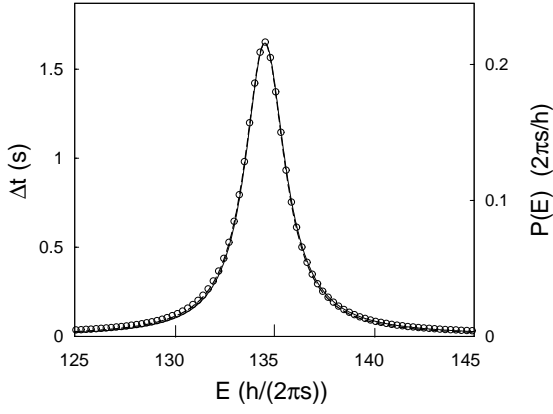


FIG. 3: Delay time  $\Delta t$  (solid line, left axis), versus incident energy of the chosen resonance (see text); the thick dashed line corresponds to a fitting with a Lorentzian profile. Energy distribution  $P(E)$  (circles, right axis) of the state which is initially the ground state for  $V_w^{Init} = 350\hbar/s$ ,  $V_b^{Init} = 400\hbar/s$ , in the final configuration  $V_w^{Fin} = 100\hbar/s$ ,  $V_b^{Fin} = 200\hbar/s$ ,  $d = 5\mu m$ ,  $b = 10\mu m$ .

this resonance we have plotted in Fig. 3 the delay time  $\Delta t$ , see Eq. (1), versus the incident energy. Also shown is the Lorentzian fitting.

If we move suddenly the bottom of the well making it shallower until the initial bound state overlaps strongly with the desired resonance (Fig. 1(b)), the wave-function will evolve in time, and the atoms will leak out through the barrier (Fig. 1(c)). We assume that the ensemble of non-interacting atoms satisfy during this process the one-dimensional time dependent Schrödinger equation,

$$i\hbar \frac{\partial \psi(x, t)}{\partial t} = \left[ -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V^{Fin}(x) \right] \psi(x, t), \quad (2)$$

where  $\psi(x, 0) = \varphi_0(x)$ . As described above, our main objective is to achieve a distribution as close as possible to the Lorentzian distribution associated with a Gamow resonance of the final potential configuration (Figs. 1(b) and 1(c)). Fig. 3 shows also the resulting total energy distribution  $P(E) = \sqrt{\frac{m}{2\hbar^2 E}} |\langle \psi_{k(E)} | \varphi_0 \rangle|^2$  of the wavepacket in the new potential configuration. ( $P(E)$  coincides with the kinetic energy distribution when the packet moves away from the potential region. [25]) After such a sudden process, several resonances are excited as it may be seen in different ways: note in particular that the Lorentzian of Fig. 3 is not normalized. This means that part of the norm is in higher resonances. A consequence is the fast decay of the non-escape probability

$$P_W(t) = \int_0^d dx |\psi(x, t)|^2$$

at short times in Fig. 4 (solid line). In other words, with the sudden switching a significant fraction of atoms is released at early times with “too much” energy. Of course, if we discard the early, fast atoms, the decay occurs finally with the desired rate and energy distribution,

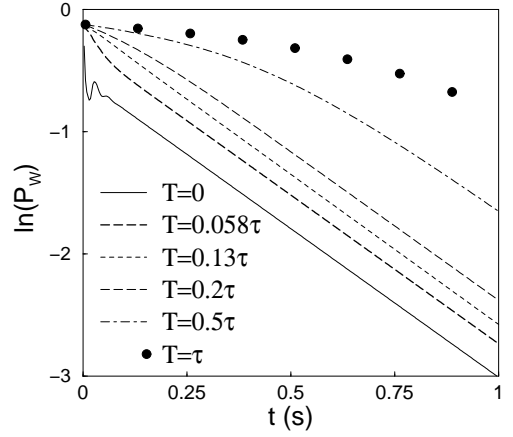


FIG. 4: Decay of the probability  $P_W(t)$  to find the atom in the well versus time for different values of  $T$ , see Eq. (3); the initial and final potential configuration are given in Fig. 3.

see Fig. 4. However, we may try to produce an ensemble without undesired high velocity components. This can be achieved by a progressive, rather than abrupt, switching of the potentials.

Let us assume that the potential profile changes in time according to the smooth function

$$V(t, x) = [V^{Fin}(x) - V^{Init}(x)](1 - e^{-t/T}) + V^{Init}(x). \quad (3)$$

The sudden change corresponds to  $T = 0$  and the infinitely slow change to  $T = \infty$ . In Fig. 4 we show the decay of the non-escape probability  $P_W(t)$ . The lifetime of the first resonance (calculated from the pole of the  $S$  matrix) is  $\tau = 0.411$  s, in perfect agreement with the fitting to the exponential decay that dominates after the early transients, independent of  $T$ . This occurs because the final potential configuration is common to all cases so that the resonance with the longest life time is the same in all cases. Nevertheless, the first transient regime varies substantially with  $T$ , and for  $T \approx \tau$  the initial decay is slowed down considerably. The best fit to the purely exponential decay is found for  $T \approx 0.13\tau$ .

Our interest is in the asymptotic and stationary energy distribution at large time,  $t_\infty \gg T$ . In Fig. 5 we have plotted the final energy distribution for different transition times  $T$ . For  $T = 0$  the distribution at the main resonance can be approximated by the normalized Breit-Wigner Lorentzian form corresponding to the pole of the selected resonance, however its height is reduced because of the excitation of higher resonances. Increasing  $T$ , an optimal value is found so that the energy distribution fits even in magnitude to the ideal Lorentzian shape. For the case studied in Fig. 5, the best fit to the Lorentzian of the selected resonance corresponds to  $T \approx 0.058\tau$ . Note that this optimal value of  $T$  is different from the one that provides the best fit to the purely exponential decay ( $T \approx 0.13\tau$ ), compare Figs. 4 and 5. As  $T$  is increased further, the distribution is deformed, the symmetry is

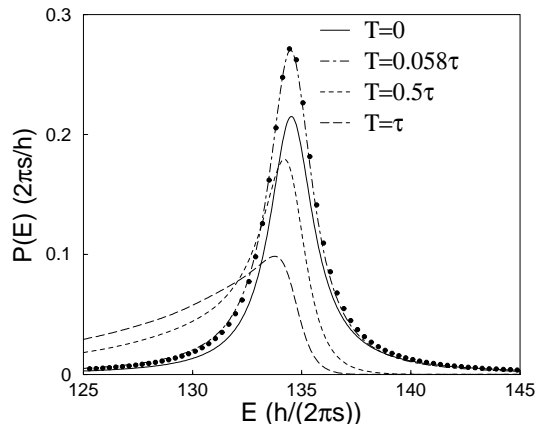


FIG. 5: Final energy distribution for different transition times  $T$ . The dots correspond to the normalized Lorentzian distribution with the parameters extracted from the  $S$ -matrix. The initial and final configuration are the same than in Fig. 3.

lost and a distortion favoring lower energies is observed.

## V. CONCLUSIONS AND DISCUSSION

We have proposed a method to prepare states with well defined average velocity and width, based on the conversion between a bound state and a resonance by changing the trapping potential. In atom optics the potentials may be realized with detuned lasers. An optimal switching time exists so that the kinetic energy distribution of the leaking atoms fits the Lorentzian form of the resonance.

Laser fluctuations may tend to broaden or blur quantum resonances. Nevertheless, stabilized lasers provide *effective constant intensities* in the time scale of  $\tau$  and  $T$  ( $\sim 0.01 - 1$  s) [20, 21, 22] so, that our analysis would apply to the effective, time-averaged potentials. Further theoretical and experimental work is required to determine the feasibility of using scattering resonances in optical traps.

Finally, the method may be applied to electrons in semiconductor heterostructures, where the well-depth is modified by potential gate voltages [23, 24].

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- [1] C. J. van den Meijdenberg in *Atomic and Molecular Beam Methods*, ed. by G. Scoles (Oxford, New York, 1988), Chap. 13.
  - [2] M. Kasevich, D. S. Weiss, E. Riis, K. Moler, S. Kasapi, and S. Chu, Phys. Rev. Lett. **66**, 2297 (1991).
  - [3] K. Moler, D. S. Weiss, M. Kasevich, and S. Chu, Phys. Rev. A **45**, 342 (1992).
  - [4] A. Aspect, E. Arimondo, R. Kaiser, N. Vansteenkiste, and C. Cohen-Tannoudji, Phys. Rev. Lett. **61**, 826 (1998).
  - [5] J. Stenger, S. Inouye, A.P. Chikkatur, D.M. Stamper-Kurn, D.E. Pritchard, and W. Ketterle, Phys. Rev. Lett. **82**, 4569 (1999).
  - [6] Yoshio Torii, Yoichi Suzuki, Mikio Kozuma, Toshiaki Sugiura, Takahiro Kuga, Lu Deng, and E.W. Hagley, Phys. Rev. A **61**, 041602(R) (2000).
  - [7] E.W. Hagley, L. Deng, M. Kozuma, J. Wen, K. Helmersson, S.L. Rolston, and W.D. Phillips, Science **283**, 1706 (1999).
  - [8] J.-Y. Courtois, G. Grynberg, B. Lounis, and P. Verkerk, Phys. Rev. Lett. **72**, 3017 (1994).
  - [9] M. Kozuma, L. Deng, E. W. Hagley, J. Wen, R. Lutwak, K. Helmersson, S. L. Rolston, and W. D. Phillips, Phys. Rev. Lett. **82**, 871 (1999).
  - [10] M. Wilkens, E. Goldstein, B. Taylor, and P. Meystre, Phys. Rev. A **47**, 2366 (1993).
  - [11] A. Ruschhaupt, F. Delgado, and J. G. Muga, J. Phys. B: At. Mol. Opt. Phys. **38**, 2665 (2005).
  - [12] L. Löffler, G. M. Meyer, and H. Walther, Europhys. Lett. **41**, 593 (1998).
  - [13] Z. M. Zhang, S. W. Xie, Y. L. Chen, Y.X. Xia, and S.K. Zhou, Phys. Rev. A **60**, 3321 (1999).
  - [14] J. Martin and T. Bastin, Eur. Phys. J. D **29**, 133 (2004).
  - [15] J.R. Angelin and W. Ketterle, Nature **416**, 211 (2002).
  - [16] K. Bongs and K. Sengstock, Rep. Prog. Phys. **67**, 907 (2004).
  - [17] T. Paul, K. Richter, and P. Schlagheck, Phys. Rev. Lett. **94**, 020404 (2005).
  - [18] I. Carusotto and G. C. La Rocca, Phys. Rev. Lett. **84**, 399 (2000).
  - [19] D. Leibfried, R. Blatt, C. Monroe, and D. Wineland, Rev. Mod. Phys. **75**, 281 (2003).
  - [20] Vincent J. Newell, F. W. Deeg, S. R. Greenfield, and M. D. Fayer, J. Opt. Soc. Am. **6**, 257 (1989).
  - [21] T. A. Savard, K. M. O'Hara, and J. E. Thomas, Phys. Rev. A **56**, R1095 (1997).
  - [22] M. J. Rodwell et al., Opt. Lett. **11**, 638 (1986).
  - [23] F. Delgado, J. G. Muga, D. G. Austing and G. García - Calderón, J. Appl. Phys. **97**, 013705 (2005); and Erratum to appear.
  - [24] F. Delgado, J. G. Muga, H. Cruz, D. Luis, and D. G. Austing, Phys. Rev. B **72**, 195318 (2005).
  - [25] This is easily derived from the intertwining relation of scattering theory, the isometry of Moller operators, and the assumption that there is no bound state in the final configuration.